Combining Propulsive and Aerodynamic Maneuvers to Achieve Optimal Orbital Transfer

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This paper presents solutions to the problem of optimal atmospheric turning of aeroassisted orbital transfer vehicles. The main purpose, however, is to demonstrate how much the required propulsive velocity change can be reduced by optimally combining aerodynamic and propulsive maneuvers during the orbital transfer. Comparisons are given for purely propulsive maneuvers, maneuvers using aerodrag only (no lift), maneuvers with all plane change occurring in the atmosphere, and maneuvers combining atmospheric and propulsive impulses optimally to achieve minimum propulsive velocity change requirements. Results are presented for vehicles with a range of maximum lift-to-drag ratios and often show substantial gains for the optimal transfer as opposed to other methods of transfer.

Nomenclature

r = radial distance to spacecraft from Earth center
V = spacecraft speed
γ = flight path angle, positive above the local horizontal
θ = longitude
ϕ = latitude
ψ = heading angle
t = time
Z = modified Chapman density variable
u = modified Chapman kinetic energy variable
s = dimensionless arc length
ρ = atmospheric density
A = spacecraft reference area
C_D = coefficient of drag
C_L = coefficient of lift
m = spacecraft mass
g = acceleration caused by gravity
σ = bank angle, positive as measured from vertically upward
E^* = maximum lift-to-drag ratio
C_L^* = coefficient of lift at maximum lift-to-drag ratio
C_D^* = coefficient of drag at maximum lift-to-drag ratio
λ = normalized lift coefficient, C_L/C_L^*
β = inverse of the atmospheric scale height
k = $\sqrt{\beta r}$
C = vertical component of the normalized lift coefficient, $\lambda \cos\sigma$
S = horizontal component of the normalized lift coefficient, $\lambda \sin\sigma$
H = Hamiltonian
F = variable defined in Eq. (18)
G = variable defined in Eq. (19)
C_f = value of C at atmospheric exit
S_f = value of S at atmospheric exit
G_f = value of G at atmospheric exit
r_1 = radius of higher orbit
r_2 = radius of lower orbit
n = r_1/r_2
μ = gravitational constant
v = dimensionless velocity, $V/V_{\text{circular at } r_2}$
i = plane-change angle
i_a = plane change occurring in the atmosphere
ϕ_j = other acute angle in a $V_i - V_j - \Delta V$ triangle besides the plane-change angle
Δv = dimensionless velocity changes
R = radius of the Earth's atmosphere from the center of the Earth
α = r_2/R
α_1 = r_1/R
V_{10} = speed in the initial circular orbit
V_{2_f} = speed in the final circular orbit
V_1 = speed when leaving the initial orbit
V_2 = speed when arriving at the second apse
V_{cR} = circular speed at the top of the atmosphere
r_{apo} = radius at apogee
x = r_{apo}/r_1

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Introduction

AEROASSISTED orbital transfer is a subject of intense research work. For the most part, analysis of optimal transfers has considered the atmospheric and space portions of the maneuver to be decoupled. This decoupling allows the propulsive and aerodynamic maneuvers to be considered separately. Extensive research work has been done on the problem of optimal atmospheric turning (e.g., Refs. 1 and 2). The research, in general, assumes that all required plane change occurs in the atmosphere and that the space impulses are used only for deorbit to the atmosphere and for reboost to a required final orbit. An excellent review of existing research work appears in Walberg.[3]

Vinh and Hanson[4] showed that by viewing the space and atmospheric maneuvers as one global optimization problem, the required velocity change is reduced. In this study, the same methods are used to extend the results of the previous paper to include varying maximum lift-to-drag ratio. All cases of transfer are between two circular orbits. The atmospheric maneuver is optimized subject to the following assumptions: no propulsive maneuvers within the atmosphere, spherical Earth, nonrotating Earth, exponential atmosphere, and parabolic drag polar. The atmospheric maneuvers are coupled with the space maneuvers in a global optimization. The equations presented in this paper are derived in Ref. 5. Results will be compared with other modes of transfer to show how much is gained in velocity change (ΔV) by making the optimal transfers.

Modes of Transfer

The intent of this study is to compare many possible methods of orbital transfer to show which are optimal, when they are optimal, and by how much. All transfers will be between circular orbits with possible plane change. The objective is to minimize ΔV with no time requirement on the

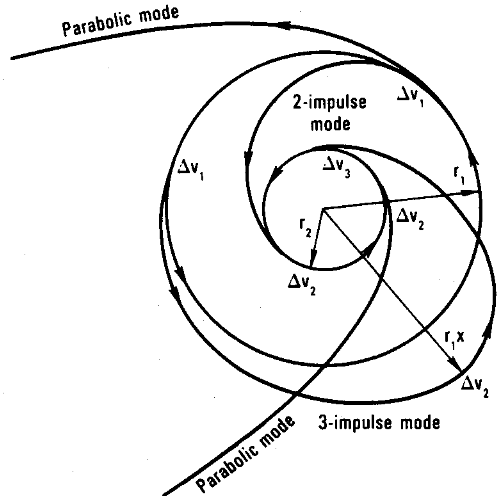

Fig. 1 Modes of optimal time-free propulsive transfer between circular orbits.

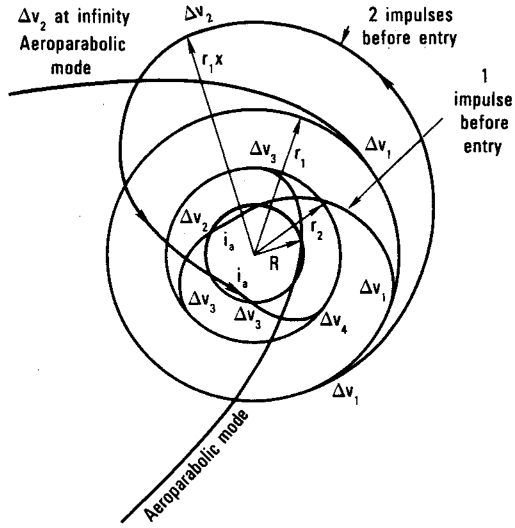

Fig. 2 Modes of optimal time-free aeroassisted transfer between circular orbits.

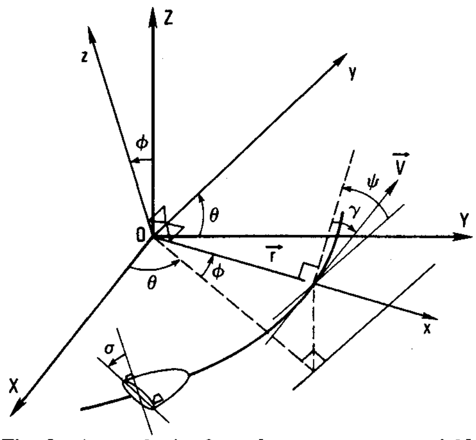

Fig. 3 Atmospheric plane-change maneuver variables.

transfer. For purposes of this discussion of transfer modes, the transfer is assumed to be from a higher orbit to a lower orbit. Transfers to a higher orbit will not yield the gains in ΔV from atmospheric maneuvers that derive from transfers to a lower orbit.

The first possible method of transfer is a totally propulsive transfer. There are three possible modes of optimal, purely propulsive transfer (Fig. 1). The first is the Hohmann mode, in which the vehicle slows down from the higher orbit so that perigee is at the lower-orbit altitude. Plane change is mixed optimally between the two propulsive impulses. The second possible mode of transfer involves increasing speed in order to transfer to a higher altitude. At apogee, the orbital transfer vehicle (OTV) slows down so that the perigee of the second perturbed orbit occurs at the final orbit's altitude. The transfer vehicle slows down into the final circular orbit, and plane change is optimally chosen between the three impulses. Also, the altitude of the second impulse is optimally chosen. The final mode is to transfer initially into a parabolic trajectory, perform all the plane change at infinity (or as far out as possible within reasonable time limits) with infinitesimal ΔV, and travel back to perigee at the lower orbit's altitude where circularization occurs. These three modes are the only possible optimal, purely propulsive modes for transfer between circular orbits with no time requirement.

The possible aeroassisted modes are analogous to the propulsive modes (Fig. 2). The first mode occurs when the spacecraft deorbits to enter the atmosphere. After exiting the atmosphere, the vehicle reboosts (if necessary) so that apogee is at the final orbit altitude, where circularization occurs. Plane change is divided optimally between the aerodynamic maneuver and the three propulsive impulses.

In the second mode, the OTV first boosts to a higher apogee and then deorbits to enter the atmosphere. Otherwise, the procedure is like the first aeroassisted mode. Plane change is divided optimally between the aerodynamic phase and the four impulses.

The final mode involves traveling on a parabolic trajectory toward infinity (or very far away, in practice), performing all the plane change there, deorbiting to enter the atmosphere, slowing down until the atmospheric exit speed enables the OTV to reach the final altitude at apogee, and recircularizing the orbit. If the plane change were performed at a large distance (not infinity), it may be optimal to perform some of the plane change within the atmosphere during speed depletion.

These three aeroassisted modes were judged to be the only ones that could be optimal. For purposes of comparison, results will be given for cases where all the plane change is performed in the atmosphere and where none of the plane change is performed in the atmosphere (drag only).

Optimal Atmospheric Turning

This section presents the equations and methods used for determination of the optimal aerodynamic maneuvers. The problem may be stated as follows: Given a vehicle with some maximum lift-to-drag ratio that enters the atmosphere and uses aerodynamic forces only while in the atmosphere, maximize the exit speed for some fixed plane-change angle. A mathematically equivalent objective is to maximize the plane-change angle for a fixed exit speed.

Figure 3 defines the variables. Details of the derivation are given in Ref. 5. If we assume a spherical, nonrotating Earth, the equations of motion are as follows:

$$\frac{dZ}{ds} = -k^2 Z \tan\gamma \tag{1}$$

$$\frac{du}{ds} = -\frac{kZu(1+C^2+S^2)}{(E^* \cos\gamma)} - (2-u)\tan\gamma \tag{2}$$

$$\frac{d\gamma}{ds} = \frac{kZC}{\cos\gamma} - \frac{1}{u} + 1 \tag{3}$$

$$\frac{d\theta}{ds} = \frac{\cos\psi}{\cos\phi} \tag{4}$$

$$\frac{d\phi}{ds} = \sin\psi \tag{5}$$

$$\frac{d\psi}{ds} = \frac{kZS}{\cos^2\gamma} - \cos\psi \tan\phi \tag{6}$$

where we have used the modified Chapman variables

$$Z = \rho A C_L^* \frac{\sqrt{r/\beta}}{2m} \tag{7}$$

$$u = \frac{V^2}{(\mu/r)} \tag{8}$$

and the independent variable is the dimensionless arc length

$$s = \int_0^t V \frac{\cos\gamma}{r} \, dt \tag{9}$$

The drag polar used in this study satisfies the relation

$$\frac{C_D}{C_D^*} = \frac{(1+\lambda^2)}{2} \tag{10}$$

where

$$\lambda = \frac{C_L}{C_L^*} \tag{11}$$

For the Earth's atmosphere, k^2 is oscillatory about a mean value

$$k^2 \simeq 900 \tag{12}$$

One may write the Hamiltonian and determine the adjoint equations from it (Ref. 5). The equations have four known integrals, with two undetermined constants plus the two additional initial values needed for integration of the adjoint equations. Thus, the problem has four parameters. Only three parameters are truly independent since the equations are linear in the adjoint variables. This problem is difficult to solve since the three constants have no physical meaning. For unbounded control,

$$\frac{\partial H}{\partial(\text{control variables})} = 0 \tag{13}$$

and therefore

$$C = \frac{E^* p_\gamma}{2u p_u} \tag{14}$$

$$S = \frac{E^* p_\psi}{2u p_u \cos\gamma} \tag{15}$$

Taking the derivatives, after some algebra, yields

$$\frac{dC}{ds} = \frac{2C(C + E^* \tan\gamma)}{E^* u} + \frac{k^2 F + E^*[(2-u)/2u] + [(kZ/2)(1 - C^2 - 3S^2) \sin\gamma}{\cos^2\gamma} \tag{16}$$

$$\frac{dS}{ds} = \frac{2SC}{E^* u} + S \tan\gamma \left(\frac{kZC}{\cos\gamma} + 1 + \frac{1}{u}\right) - S \tan\phi \sin\psi - \frac{G \cos\psi}{\cos\gamma} \tag{17}$$

where

$$F = \frac{E^* Z p_z}{2u p_u} \tag{18}$$

and

$$G = \frac{E^* p_\phi}{2u p_u} \tag{19}$$

Taking the derivatives of F and G yields

$$\frac{dF}{ds} = \frac{2F(C + E^* \tan\gamma)}{E^* u} + \frac{kZ(1 - C^2 - S^2)}{2 \cos\gamma} \tag{20}$$

and

$$\frac{dG}{ds} = \frac{2G(C + E^* \tan\gamma)}{E^* u} + \frac{S \cos\gamma \cos\psi}{\cos^2\phi} \tag{21}$$

Now the 10 equations to be integrated are the state equations and the differential equations for C, S, F, and G. Since the Hamiltonian is equal to zero (free final time),

$$-k^2 F \tan\gamma - \frac{E^*(2-u)\tan\gamma}{2u} - \frac{kZ(1 - C^2 - S^2)}{2\cos\gamma} - S \cos\gamma \cos\psi \tan\phi + C[1 - (1/u)] + G \sin\psi = 0 \tag{22}$$

If we use Eq. (22) to solve for one parameter, the problem now has three parameters, where two of the initial values (C and S) may be estimated by physical arguments. This Hamiltonian integral can be used to replace one of the four adjoint equations, but it is more convenient to integrate all the equations and use Eq. (22) to check the accuracy of the numerical solution.

To maximize the final speed for a fixed plane-change angle, the transversality conditions are

$$C_f = 0 \tag{23}$$

and

$$\frac{G_f}{S_f} = \frac{\tan\phi_f \cos\gamma_f}{\tan\psi_f} \tag{24}$$

The atmospheric plane-change angle i_a is given by

$$\cos i_a = \cos\phi_f \cos\psi_f \tag{25}$$

where the initial values for θ, ϕ, and ψ are 0. Integrating the equations numerically with the proper choice of initial conditions yields Figs. 4, 5, and 6 for varying maximum lift-to-drag ratio. Johannesen[6] studied the effect of varying maximum lift-to-drag ratio on atmospheric flight, but did not extend the results to subcircular atmospheric exit speeds as is the case here.

Equations for Optimal Propulsive Orbit Transfer

Next, the equations for optimization and Δv for the cases of purely propulsive transfers are presented. The paper includes

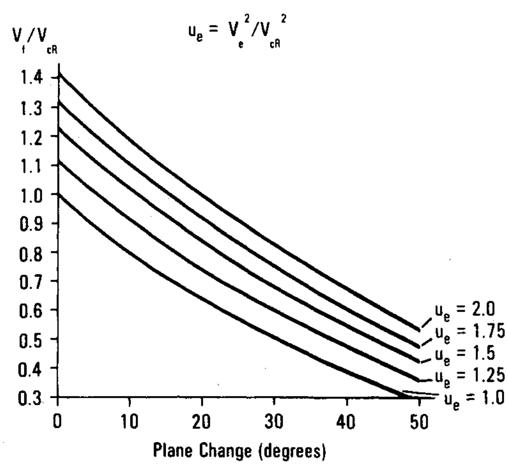

Fig. 4 Final dimensionless velocity vs entry speed V_e/V_{cR}. Maximum lift-to-drag ratio = 1.0 $u_e = V_e^2/V_{cR}^2$.

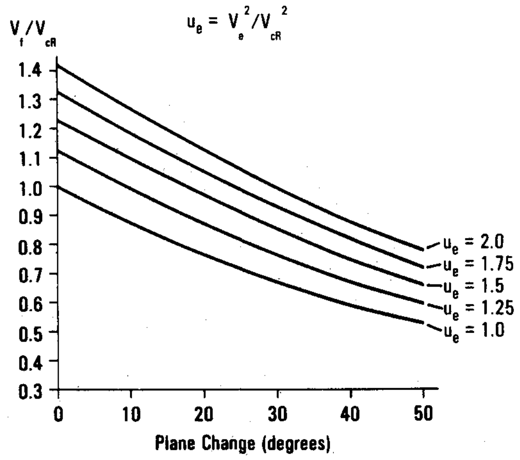

Fig. 5 Final dimensionless velocity vs entry speed V_e/V_{cR}. Maximum lift-to-drag ratio = 1.5 $u_e = V_e^2/V_{cR}^2$.

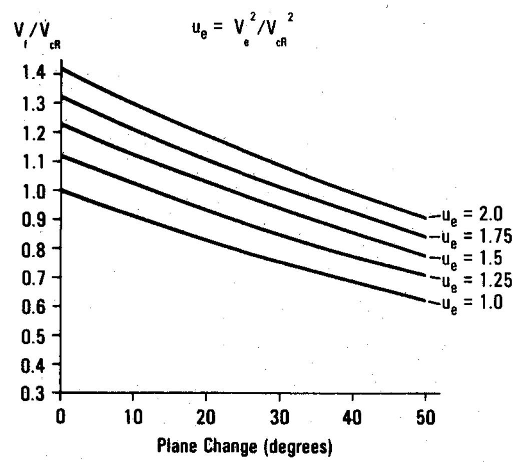

Fig. 6 Final dimensionless velocity vs entry speed V_e/V_{cR}. Maximum lift-to-drag ratio = 2.0 $u_e = V_e^2/V_{cR}^2$.

all equations for purposes of completeness. Derivations are in Ref. 5. Let

$$n = \frac{r_1}{r_2} \tag{26}$$

where r_1 is the radius of the higher orbit and r_2 the radius of the lower orbit. Dimensionless velocities are defined as

$$v_i = \frac{V_i}{\sqrt{\mu/r_2}} \tag{27}$$

and

$$\Delta v_i = \frac{\Delta V_i}{\sqrt{\mu/r_2}} \tag{28}$$

For the parabolic transfer, we have

$$\Delta v_p = (\sqrt{2}-1)\left(1+\frac{1}{\sqrt{n}}\right) \tag{29}$$

For the modified Hohmann transfer, the optimization equations are

$$\frac{\sin(\phi_1+i_1)}{\sin\phi_1} = \frac{V_{10}}{V_1} \tag{30}$$

$$\frac{\sin(\phi_2+i_2)}{\sin\phi_2} = \frac{V_2}{V_{2f}} \tag{31}$$

$$i_1 + i_2 = i \tag{32}$$

and

$$\sin\phi_1 - \sqrt{n}\,\sin(\phi_2+i_2) = 0 \tag{33}$$

to be solved for i_1, i_2, ϕ_1, and ϕ_2. The velocity changes are given by

$$\Delta v_1 = \frac{\sin i_1}{[\sqrt{n}\,\sin(i_1+\phi_1)]} \tag{34}$$

and

$$\Delta v_2 = \frac{\sin i_2}{\sin\phi_2} \tag{35}$$

For the three-impulse transfer, the optimization equations are

$$\frac{\sin(\phi_1+i_1)}{\sin\phi_1} = \sqrt{\frac{2x}{(x+1)}} \tag{36}$$

$$\frac{\sin(\phi_2+i_2)}{\sin\phi_2} = \sqrt{\frac{1+nx}{1+x}} \tag{37}$$

$$\frac{\sin(\phi_3+i_3)}{\sin\phi_3} = \sqrt{\frac{2nx}{1+nx}} \tag{38}$$

$$i_1+i_2+i_3 = i \tag{39}$$

$$x\,\sin\phi_1 - \sin\phi_2 = 0 \tag{40}$$

$$nx\sqrt{\frac{1+x}{1+nx}}\,\sin\phi_3 - \sin\phi_2 = 0 \tag{41}$$

$$\frac{n\cos\phi_3}{nx+1} - \frac{(2nx+1)\sin i_2}{x(nx+1)\sin\phi_2} + \frac{\cos\phi_1\sqrt{nx+1}}{(1+x)^{3/2}} + \frac{\cos\phi_2(n-1)}{(1+x)^{3/2}\sqrt{nx+1}} = 0 \tag{42}$$

to be solved for the seven parameters i_1, i_2, i_3, ϕ_1, ϕ_2, ϕ_3, and x, where x is defined such that the highest distance r_{apo} is given by

$$r_{apo} = r_1 x \tag{43}$$

The required velocity changes are given by

$$\Delta v_1 = \frac{\sin i_1}{\sqrt{n}\,\sin\phi_1} \tag{44}$$

$$\Delta v_2 = \frac{\sin i_2}{\sin\phi_2}\sqrt{\frac{2}{nx(nx+1)}} \tag{45}$$

$$\Delta v_3 = \frac{\sin i_3}{\sin\phi_3} \tag{46}$$

Equations for Optimal Aeroassisted Transfer

Let

$$\alpha = \frac{r_2}{R} \tag{47}$$

where R is the radius to the top of the atmosphere. Then, for the aeroassisted parabolic mode,

$$\Delta v_{AP} = \frac{\sqrt{2}-1}{\sqrt{n}} + 1 - \sqrt{\frac{2}{\alpha+1}} \tag{48}$$

Now for the aeroassisted mode with one impulse before entry, let

$$\alpha_1 = \frac{r_1}{R} \tag{49}$$

$$i_{\text{TOT}} = \text{total required plane change} \quad (50)$$

$$i_a = \text{atmospheric plane change} \quad (51)$$

$$i_p = i_{\text{TOT}} - i_a = \text{propulsive plane change} \quad (52)$$

and

$$i_j = \text{plane change occurring at } j\text{th impulse} \quad (53)$$

The optimization equations are

$$\frac{1}{\sqrt{n}}\sqrt{\frac{1}{\alpha_1+1}}\sin i_1 \,\Delta v_2 - \alpha\sqrt{\frac{\alpha_1}{\alpha+1}}\Delta v_1 \frac{V_f}{V_{cR}} \times \sin(i_p - i_1 - i_3) = 0 \quad (54)$$

and

$$\sin i_3 \,\Delta v_2 - \Delta v_3 \alpha\sqrt{\alpha}V_f/V_{cR}\sin(i_p - i_1 - i_3) = 0 \quad (55)$$

where the velocity changes are given by

$$\Delta v_1 = \frac{1}{\sqrt{n}}\left[\frac{\alpha_1+3}{\alpha_1+1} - 2\sqrt{\frac{2}{\alpha_1+1}}\cos i_1\right]^{1/2} \quad (56)$$

$$\Delta v_2 = \left[\frac{2\alpha^2}{\alpha+1} + \alpha\left(\frac{V_f}{V_{cR}}\right)^2 - 2\alpha\sqrt{\frac{2\alpha}{\alpha+1}}\frac{V_f}{V_{cR}} \times \cos(i_p - i_1 - i_3)\right]^{1/2} \quad (57)$$

and

$$\Delta v_3 = \left[\frac{\alpha+3}{\alpha+1} - 2\sqrt{\frac{2}{\alpha+1}}\cos i_3\right]^{1/2} \quad (58)$$

For cases where Δv_2 and i_2 are optimally zero, Eq. (55) may be replaced by

$$\frac{1}{\sqrt{n}}\sqrt{\frac{1}{\alpha_1+1}}\sin i_1 \,\Delta v_3 - \sin i_3 \,\Delta v_1 \frac{\sqrt{n}}{\sqrt{\alpha+1}} = 0 \quad (59)$$

For details of the solution, see Ref. 5. Iterating over possible i_a and solving for the optimum atmospheric and propulsive maneuvers in each case yields the global optimum.

For the case of aeroassisted transfer with two impulses before atmospheric entry, there are two possibilities. If the speed at atmospheric exit is sufficient to reach the required apogee, then $\Delta v_3 = 0$. In this case,

$$\sin i_1 = \frac{1}{x}\sqrt{\frac{2}{(x\alpha_1+1)x}}\sin(i_p - i_1)\frac{\Delta v_1}{\Delta v_2} \quad (60)$$

for the optimization since the plane change i_4 is very close to zero. Otherwise, one has the three equations

$$\sin i_4 \,\Delta v_3 - \alpha\sqrt{\alpha}\frac{V_f}{V_{cR}}\sin i_3 \,\Delta v_4 = 0 \quad (61)$$

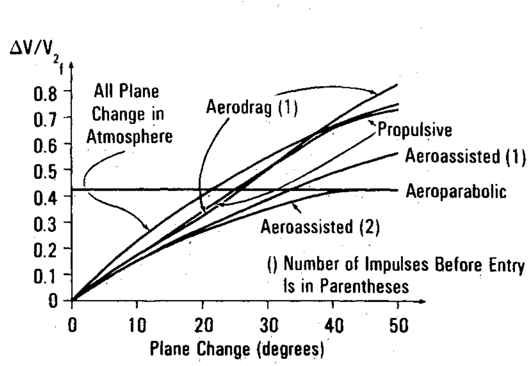

Fig. 7 Optimal transfer, $n = 1.0$. Maximum lift-to-drag ratio = 1.0.

$$\alpha\sqrt{\frac{\alpha}{\alpha+1}}\frac{V_f}{V_{cR}}\sin i_3 \,\Delta v_1 - \frac{1}{n}\sqrt{\frac{x}{x+1}}\sin i_1 \,\Delta v_3 = 0 \quad (62)$$

$$\alpha\sqrt{\frac{\alpha}{\alpha+1}}\frac{V_f}{V_{cR}}\sin i_3 \,\Delta v_2 - \frac{1}{nx}\sqrt{\frac{1}{(x+1)(x\alpha_1+1)}} \times \sin(i_p - i_1 - i_3 - i_4)\,\Delta v_3 = 0 \quad (63)$$

One solves the foregoing equations for a given value of x and then iterates over all possible x values, modifying the atmospheric flight accordingly, to search for the global optimum. The velocity changes are given by

$$\Delta v_1 = \left[\left(1 + \frac{2x}{x+1} - 2\sqrt{\frac{2x}{x+1}}\cos i_1\right)\frac{1}{n}\right]^{1/2} \quad (64)$$

$$\Delta v_2 = \left\{\left[\frac{2}{[x(x+1)]} + \frac{2}{[x(x\alpha_1+1)]} - \frac{4}{x}\sqrt{1/[(x+1)(x\alpha_1+1)]} \times \cos(i_p - i_1 - i_3 - i_4)\right\} \quad (65)$$

$$\Delta v_3 = \left[\frac{2\alpha^2}{\alpha+1} + \alpha\left(\frac{V_f}{V_{cR}}\right)^2 - 2\alpha\sqrt{\frac{2\alpha}{\alpha+1}}\frac{V_f}{V_{cR}}\cos i_3\right]^{1/2} \quad (66)$$

$$\Delta v_4 = \left[\frac{\alpha+3}{\alpha+1} - 2\sqrt{\frac{2}{\alpha+1}}\cos i_4\right]^{1/2} \quad (67)$$

Results

The purpose of this study is to show which modes of transfer are optimal and how much velocity change can be saved by using the optimal transfer. The following results contain nine graphs showing the Δv for orbital transfer vehicles with varying lift capability and various initial and final orbits. After solving the equations for optimal atmospheric flight, one chooses an atmospheric plane change and solves for the optimal exoatmospheric (propulsive) flight. The procedure is to then iterate over atmospheric plane change to determine the global optimum for the orbital transfer.

In the figures, n is the radius ratio r_1/r_2, where r_1 is the higher (initial) orbit radius, and r_2 the final orbit radius. The

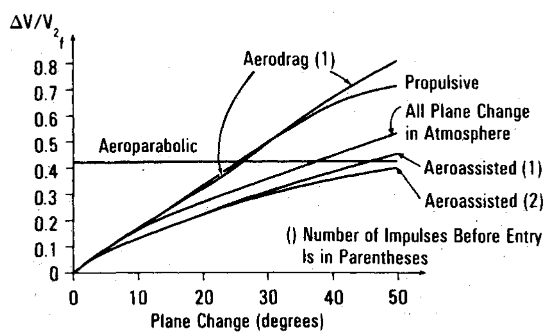

Fig. 8 Optimal transfer, $n = 1.0$. Maximum lift-to-drag ratio = 1.5.

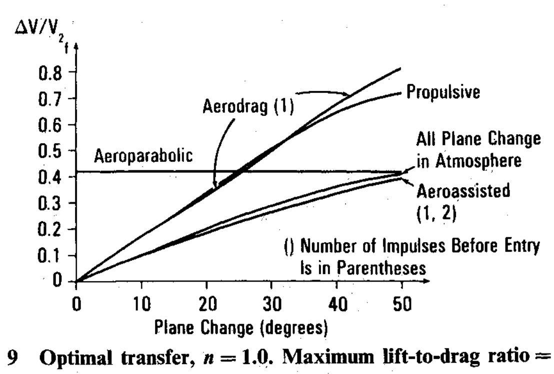

Fig. 9 Optimal transfer, $n = 1.0$. Maximum lift-to-drag ratio = 2.0.

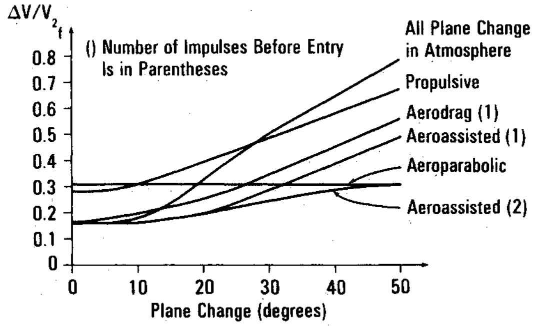

Fig. 10 Optimal transfer, $n = 2.0$. Maximum lift-to-drag ratio = 1.0.

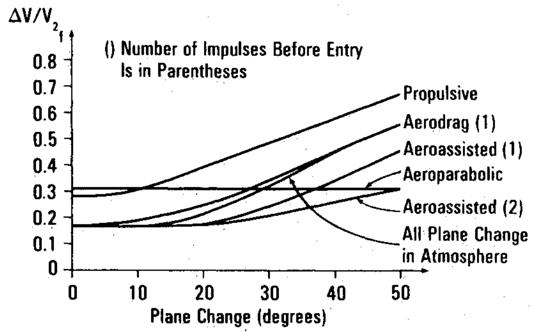

Fig. 11 Optimal transfer, $n = 2.0$. Maximum lift-to-drag ratio = 1.5.

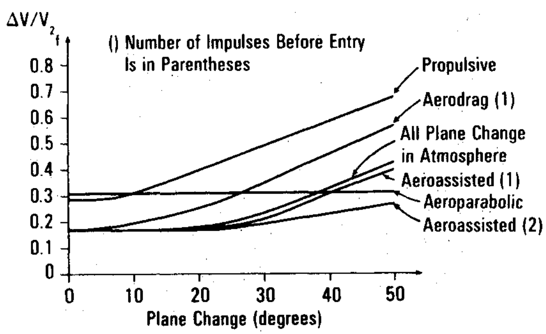

Fig. 12 Optimal transfer, $n = 2.0$. Maximum lift-to-drag ratio = 2.0.

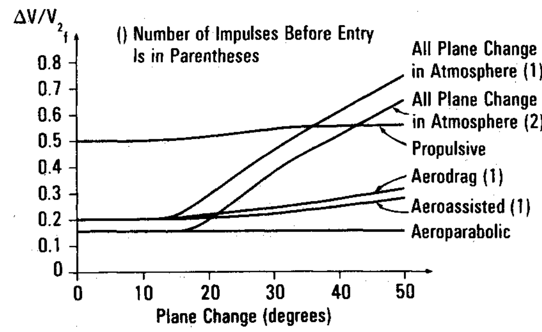

Fig. 13 Optimal transfer, $n = 6.365$. Maximum lift-to-drag ratio = 1.0.

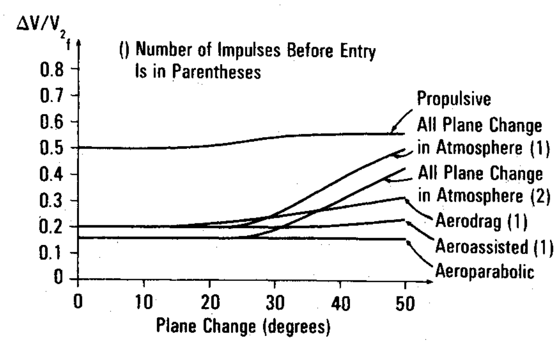

Fig. 14 Optimal transfer, $n = 6.365$. Maximum lift-to-drag ratio = 1.5.

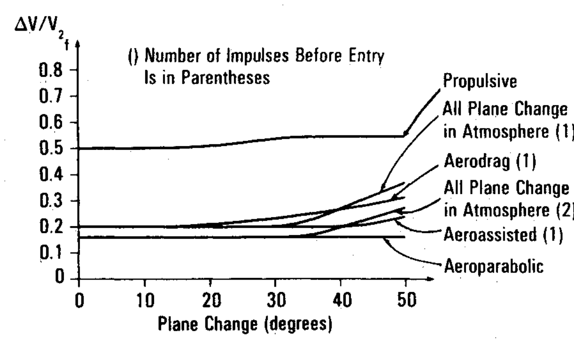

Fig. 15 Optimal transfer, $n = 6.365$. Maximum lift-to-drag ratio = 2.0.

ratio of the final orbit radius to the radius of the top of the atmosphere is 1.04 in all cases, so that the final orbit is a low-Earth orbit. Figures 7–9 give results for plane change between low-Earth orbits ($n = 1$), whereas Figs. 10–12 give results for medium-to-low transfer ($n = 2$), and Figs. 13–15 give results for transfer from geosynchronous altitude to low-Earth orbits ($n = 6.365$). Each figure shows the necessary velocity change (Δv) as a function of plane-change angle. The ordinate is dimensionless velocity change, the total velocity change divided by circular velocity at the final orbit radius. The figures give three cases of maximum lift-to-drag ratio: 1.0, 1.5, and 2.0.

Figure 7 shows the case where the initial and final orbits are low, with plane change only required (no altitude change). First, for small plane-change angles, the aerodrag-only mode yields no improvement over the propulsive mode of transfer. Above approximately 26 deg, substantial Δv is saved over the totally propulsive mode with aerodrag by first traveling out very far (aeroparabolic).

The method of doing all the plane change in the atmosphere increases the required velocity change, because of the low E^* (maximum lift-to-drag ratio) of 1.0. Note that in contrast to the case for $n = 6.365$ (Fig. 13), two impulses before atmospheric entry do not improve the Δv when all plane change is performed in the atmosphere.

Next, consider the optimal atmospheric plane change. Define "free" plane change to be that plane change that causes a final velocity at atmospheric exit just sufficient to enable the spacecraft to arrive at apogee at the desired final orbit altitude. In the case of one impulse before entry, the optimal atmospheric plane change is greater than the free plane change (free plane change is zero for the case of $n = 1$).

However, for larger plane-change angles, it is more advantageous to use two impulses before entry and take only the free plane change in the atmosphere. There are two advantages of the two-impulses-before-entry mode. A higher altitude allows part of the plane change to be performed with lower fuel expenditure at the higher altitude, and the higher altitude gives a higher entry speed, yielding extra free atmospheric plane change.

In Fig. 8, all aeroassisted modes with atmospheric plane change use less fuel than in Fig. 7 because of the increased E^*. The aeroassisted mode with one to two impulses before entry again gives the smallest Δv. The case of all plane change in the atmosphere now appears much more favorable. Figure 9 continues the same effect, with the result that an E^* of 2 implies one would always like to perform plane change in the atmosphere. Performing all plane change in the atmosphere does not significantly increase the Δv requirement since the OTV is so efficient an aerodynamic vehicle. Figures 10–15 display similar results for different orbit transfers. Although aeroassisted transfers use substantially less Δv, a system tradeoff would need to include the additional weight associated with the aeroassisted spacecraft.

In all cases considered, the aeroassisted transfers with plane change mixed optimally between propulsive and atmospheric maneuvers yielded a lower Δv expenditure than the other modes of transfer (propulsive, aerodrag, and all plane change in the atmosphere). Also, whenever there are multiple impulses, the minimum Δv maneuver involves performing some of the plane change at each impulse.

In the case of the aeroassisted transfer with two impulses before entry, one may obtain an estimate of the radius of the second impulse from the graphs. If the aeroparabolic mode is optimal, the second impulse should be performed at infinity (e.g., Fig. 13). When the aeroassisted (2) transfer is optimum, the optimal radius of the second impulse approaches infinity as the curve approaches aeroparabolic transfer (e.g., Fig. 10). The optimal radius for the second impulse approaches the initial orbit radius as this same curve approaches the aeroassisted (1) curve.

Now consider the case of all plane change in the atmosphere. For the case of transfer from geosynchronous orbit (Figs. 13–15), one always obtains a lower Δv by using two impulses before entry, and the second impulse is always at infinity. In order to demonstrate that the OTV does not really need to travel to infinity to realize the velocity savings, consider the case of $n = 6.365, E^* = 2.0$, plane change = 50 deg. If the radius at the second impulse is x multiplied by the radius at the first impulse, an x of 1 gives a Δv of about 0.37. For $x = 2, \Delta v = 0.33$, whereas an x of 10 gives $\Delta v = 0.29$. At an x of 100, the Δv is 0.277, whereas at $x = 1000$ the Δv has only decreased to 0.276. Whenever the required plane change is all free, the case of all plane change in the atmosphere reduces to another case (e.g., Fig. 15, where the free plane change is > 30 deg). For other values of n, there are some regions where two impulses and travel to infinity are the best combination, some regions where one impulse before entry is best, and some regions where an intermediate-altitude second impulse is desirable.

In the case of aerodrag, two impulses before entry without travel to infinity are rarely optimal and save < 0.02 on Δv when they are optimal. In all other cases, the optimal aerodrag mode is either one impulse before entry or the aeroparabolic mode.

The only case in which "forced" ($>$ free) atmospheric plane change was found to be optimal and to yield any significant Δv savings was for the $n = 1, E^* = 2$ case. This makes sense because there is no free atmospheric plane change and the vehicle is very aerodynamically efficient. The amount of Δv savings, for example, is 0.15 for a plane change of 20 deg.

Summary

This paper presents analysis of optimal aeroassisted transfer between two circular orbits. By treating the space and atmospheric portions of the transfer as a single global optimization problem, one may find the lowest Δv transfers. This paper gives equations for the optimal space and atmospheric transfers and combines the results to achieve the global optimum. Results are given for transfers from various orbits to low-Earth orbits with varying plane-change angles and several values of maximum lift-to-drag ratio. Results show that the optimal aeroassisted transfer usually requires a significantly smaller Δv than a totally propulsive transfer, a transfer using aerodrag only, or a transfer involving performance of all required plane change in the atmosphere.

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